F07ARF (CGETRF/ZGETRF) – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

F07ARF (CGETRF/ZGETRF) computes the LU factorization of a complex m by n matrix.

2 Specification

```
SUBROUTINE F07ARF(M, N, A, LDA, IPIV, INFO)
ENTRY      cgetrf(M, N, A, LDA, IPIV, INFO)
INTEGER           M, N, LDA, IPIV(*), INFO
complex           A(LDA,*)
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine forms the LU factorization of a complex m by n matrix A as $A = PLU$, where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). Usually A is square ($m = n$), and both L and U are triangular. The routine uses partial pivoting, with row interchanges.

4 References

[1] Golub G H and van Loan C F (1996) *Matrix Computations* Johns Hopkins University Press (3rd Edition), Baltimore

5 Parameters

1: M — INTEGER *Input*

On entry: m, the number of rows of the matrix A.

Constraint: M ≥ 0.

2: N — INTEGER *Input*

On entry: n, the number of columns of the matrix A.

Constraint: N ≥ 0.

3: A(LDA,$*$) — ***complex*** array *Input/Output*

Note: the second dimension of the array A must be at least max(1,N).

On entry: the m by n matrix A.

On exit: A is overwritten by the factors L and U; the unit diagonal elements of L are not stored.

4: LDA — INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07ARF (CGETRF/ZGETRF) is called.

Constraint: LDA $\geq$ max(1,M).

5: IPIV(∗) — INTEGER array *Output*

Note: the dimension of the array IPIV must be at least max(1,min(M,N)).

On exit: the pivot indices. Row i of the matrix A was interchanged with row IPIV(i), for $i = 1, 2, \ldots, \min(m, n)$.

6: INFO — INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i, $u_{i,i}$ is exactly zero. The factorization has been completed but the factor U is exactly singular, and division by zero will occur if it is subsequently used to solve a system of linear equations or to compute A^{-1}.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^3$ if $m = n$ (the usual case), $\frac{4}{3}n^2(3m - n)$ if $m > n$ and $\frac{4}{3}m^2(3n - m)$ if $m < n$.

A call to this routine with $m = n$ may be followed by calls to the routines:

F07ASF (CGETRS/ZGETRS) to solve $AX = B$, $A^TX = B$ or $A^HX = B$;

F07AUF (CGECON/ZGECON) to estimate the condition number of A;

F07AWF (CGETRI/ZGETRI) to compute the inverse of A.

The real analogue of this routine is F07ADF (SGETRF/DGETRF).

9 Example

To compute the LU factorization of the matrix A, where

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}.$$

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F07ARF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MMAX, NMAX, LDA
      PARAMETER        (MMAX=8,NMAX=8,LDA=MMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, M, N
*     .. Local Arrays ..
      complex          A(LDA,NMAX)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         cgetrf, X04DBF
*     .. Intrinsic Functions ..
      INTRINSIC        MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07ARF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) M, N
      IF (M.LE.MMAX .AND. N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) ((A(I,J),J=1,N),I=1,M)
*
*        Factorize A
*
         CALL cgetrf(M,N,A,LDA,IPIV,INFO)
*
*        Print details of factorization
*
         WRITE (NOUT,*)
         IFAIL = 0
         CALL X04DBF('General',' ',M,N,A,LDA,'Bracketed','F7.4',
     +               'Details of factorization','Integer',RLABS,
     +               'Integer',CLABS,80,0,IFAIL)
*
*        Print pivot indices
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'IPIV'
         WRITE (NOUT,99999) (IPIV(I),I=1,MIN(M,N))
*
         IF (INFO.NE.0) WRITE (NOUT,*) 'The factor U is singular'
*
      END IF
      STOP
*
99999 FORMAT ((1X,I12,3I18))
      END
```

9.2 Program Data

```
F07ARF Example Program Data
  4  4                                                    :Values of M and N
 (-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
 (-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
 (-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
 ( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67)  :End of matrix A
```

9.3 Program Results

```
F07ARF Example Program Results

Details of factorization
                    1                 2                 3                 4
1  (-3.2900,-2.3900) (-1.9100, 4.4200) (-0.1400,-1.3500) ( 1.7200, 1.3500)
2  ( 0.2376, 0.2560) ( 4.8952,-0.7114) (-0.4623, 1.6966) ( 1.2269, 0.6190)
3  (-0.1020,-0.7010) (-0.6691, 0.3689) (-5.1414,-1.1300) ( 0.9983, 0.3850)
4  (-0.5359, 0.2707) (-0.2040, 0.8601) ( 0.0082, 0.1211) ( 0.1482,-0.1252)

IPIV
           3                 2                 3                 4
```